1. For an arbitrary normalized state $|\Psi\rangle$ and operator $\hat{A}$, calculate the sum $\sum_i |\langle \Psi | \hat{A} | k_i \rangle|^2$ over the complete basis $\{|k_i\}\}$. What value is obtained if $\hat{A}$ is unitary?

2. Show by the use of bra-ket notation that

$$\text{Trace}(\hat{A}) = \sum_i \langle k_i | \hat{A} | k_i \rangle$$

is independent of the choice of basis $\{|k_i\}\}$.  

3. Atkins/Friedman, problem 1.11.

4. Find the eigenvectors common to both matrices below and show that by switching to the basis of common eigenvectors, both matrices are diagonalized.

\[
A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}
\]

\[
B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 2 \end{bmatrix}
\]