CHEM 6491: Quantum Mechanics
Problem Set I
Due Tuesday, September 3

1. Atlanta Braves pitcher John Smoltz can throw a 95 mph fastball. A regulation baseball must weigh between 5 and 5.25 ounces. What is the wavelength of a John Smoltz fastball weighing 5 ounces? (1 lb = 16 oz). Could a researcher detect any wave-like properties (like interference) for fastballs?

2. In Bohr’s model of the hydrogen atom, electrons travel around the nucleus in certain stable orbits of fixed radius $r$. In this case, the centripetal force $\frac{mv^2}{r}$ which keeps the electron from flying away is supplied by the Coulomb attraction, $\frac{e^2}{4\pi\varepsilon_0 r^2}$. Note that we will assume the electron is orbiting around a fixed nucleus; since the proton is much more massive than the electron (by about 2000 times), this is a good approximation. Otherwise, we could use a reduced mass.

   (a) Equate the centripetal force with the Coulomb force, use the fact that only certain angular momenta are allowed (Bohr’s assumption $l = mvr = n\hbar$), and solve for the allowed Bohr orbits $r$.

   (b) The energy of the hydrogen atom is given by a sum of kinetic and potential energies,

   $$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\varepsilon_0 r}.$$  \hspace{1cm} (1)

   Again using the relation between the centripetal force and the Coulomb force, rewrite $E$ in terms of $r$ and physical constants only (i.e., eliminate the $mv^2$ term) and then in the place of $r$ substitute the expression (solved for above) for the allowed orbits. This should give the allowed energy levels.

   (c) Using the fact that a transition between two allowed energy levels is the same as the energy of the photon emitted/absorbed $\Delta E = h\nu$, derive the Rydberg formula for the H atom spectrum,

   $$\Delta E = \frac{me^4}{8\varepsilon_0^2\hbar^2} \left( \frac{1}{n^2} - \frac{1}{m^2} \right) = h\nu,$$  \hspace{1cm} (2)

   where $n$ and $m$ are two integers with $n < m$.

3. Which of the following sets of vectors is linearly independent?

   (a) $6\hat{i} + 3\hat{j}, -2\hat{i} - \hat{j}$

   (b) $\hat{i} + 2\hat{k}, \hat{j}, \hat{i} + \hat{j} + \hat{k}$

   (c) $\hat{i} + \hat{j}, -\hat{i} + \hat{j}, \hat{k}$
(d) \(|\cos(\omega t)|, |2\cos(\omega t)|\)

(e) \[
\begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 1 \\
2 & 0
\end{bmatrix}
\]

4. Apply the Gram-Schmidt procedure to \[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}, \begin{bmatrix}
1 \\
0 \\
1
\end{bmatrix}, \begin{bmatrix}
2 \\
1 \\
0
\end{bmatrix}
\] to obtain three orthonormal vectors. Begin with the first vector given.