Introduction to Semiempirical Methods

C. David Sherrill

School of Chemistry and Biochemistry
Georgia Institute of Technology
Semiempirical Methods

- Approximate version of Hartree-Fock
- Some two-electron and sometimes one-electron integrals are neglected to speed up the computation
- Some empirical parameters are inserted to make up for the neglected integrals
Eliminating Integrals

- Hartree-Fock formally scales as $O(N^4)$ because of the two-electron integrals (although this approaches $O(N^2)$ with Schwarz screening, and ultimately $O(N)$ for large molecules with multipole methods)

- Semiempirical methods reduce cost by eliminating many integrals

Atomic orbital $\mu_A$ is centered on atom A, etc.
Approximations in Semiempirical Methods

- Only valence electrons considered: core is treated by reducing nuclear charge or by adding special core functions.
- Only a minimum basis is used (one AO basis function per real AO in atom): usually Slater functions are used!
- Various approximations for one- and two-electron integrals.
Zero Differential Overlap (ZDO)

- ZDO is a common approximation to many semi-empirical methods
- Set the product $\mu_A(r)\nu_B(r) = 0$ for $\mu, \nu$ on different atoms ($A \neq B$)
- For $(\mu\nu|\lambda\sigma)$ this means all 3 & 4 center 2-electron integrals (most numerous ones) are neglected
- Get away with this by introducing parameters into (some of the) remaining integrals
Neglect of Diatomic Differential Overlap (NDDO)

- Semiempirical method based on ZDO
- Nuclear charge reduced by # of core electrons
- For two-electron integrals, 
  \[(\mu_A \nu_B | \lambda_C \sigma_D) = \delta_{AB} \delta_{CD} (\mu_A \nu_B | \lambda_C \sigma_D)\] (i.e., ZDO)
- For one-electron integrals, neglect those involving 3 centers (see next page)
Neglect of 3-center 1-electron Integrals in NDDO

\[ \hat{h} = -\frac{1}{2} \nabla^2 - \sum_A \frac{Z'_A}{|\vec{R}_A - \vec{r}|} = -\frac{1}{2} \nabla^2 - \sum_A V_A \]

\[ \langle \mu_A | \hat{h} | \nu_B \rangle = \langle \mu_A | -\frac{1}{2} \nabla^2 - V_A | \nu_A \rangle - \sum_{B \neq A} \langle \mu_A | V_B | \nu_A \rangle \]

\[ \langle \mu_A | \hat{h} | \nu_B \rangle = \langle \mu_A | -\frac{1}{2} \nabla^2 - V_A - V_B | \nu_B \rangle \]

\[ \langle \mu_A | V_C | \nu_B \rangle = 0 \]
Intermediate Neglect of Differential Overlap (INDO)

- Similar to NDDO, but also neglect two-center integrals in which the AO’s on an atom are different; only keep $(\mu_A\mu_A|\nu_B\nu_B)$ 2-center ints and $(\mu_A\nu_A|\lambda_A\sigma_A)$ 1-center ints.
- Same approximations for 1-electron integrals as NDDO.
Complete Neglect of Differential Overlap (CNDO)

- Most drastic of the NDDO, INDO, CNDO series
- Only keep two-electron integrals of the type
  \((\mu_A \mu_A | \nu_B \nu_B)\) 2-center ints and
  \((\mu_A \mu_A | \nu_A \nu_A)\) 1-center ints
Parameterization of Semiempirical Methods

For integrals that are kept, some of them are computed exactly, and others are computed using parameters from experiment (semi-empirical)
Modified Intermediate Neglect of Differential Overlap (MINDO)

\[
\langle \mu_A | \hat{h} | \nu_B \rangle = \langle \mu_A | -\frac{1}{2} \nabla^2 - V_A | \nu_A \rangle - \sum_{B \neq A} \langle \mu_A | V_B | \nu_A \rangle
= \langle \mu_A | \nu_B \rangle \beta_{AB} (I_\mu + I_\nu)
\]

- $I_\mu$ is the ionization potential of an electron in atomic orbital $\mu$
- $\beta_{AB}$ is a fitted “diatomic” parameter
Modified Intermediate Neglect of Differential Overlap (MINDO)

MINDO has been largely superseded by NDDO models based on atomic parameters, such as MNDO, AM1, PM3, which differ only in their treatment of core and how parameters are assigned.

\[
\langle \mu_A | \hat{h} | \nu_B \rangle = \langle \mu_A | -\frac{1}{2} \nabla^2 - V_A | \nu_A \rangle - \sum_{B \neq A} \langle \mu_A | V_B | \nu_A \rangle
\]

\[
= \langle \mu_A | \nu_B \rangle \frac{1}{2} (\beta_\mu + \beta_\nu)
\]
Modified Neglect of Diatomic Overlap (MNDO)

- Atomic parameterization, modified NDDO type model
- Largely superseded by AM1, PM3, etc.
- Some limitations:
  - Steric crowding exaggerated
  - H-bonds not reliable
  - Hypervalent molecules too unstable
  - Bond breaking/forming TS’s too high in energy
Austin Model 1 (AM1)

- Changed core functions relative to MNDO, reparameterized
- Improved over MNDO:
  - Alkyl groups too stable by ~2 kcal/mol per CH$_2$
  - Peroxide compounds not described well

MNDO Parametric Method 3 (PM3)

- Reoptimized parameters automatically (previously done by hand) and changed core terms again
- Quite a few atoms available
- Performance:
  - H-bonds too short by ~0.1 Å
  - N atom charges often unreasonable

Common Problems for MNDO, AM1, PM3

- Rotations about partial double bonds give barriers that are too low
- Parameters for metals based on only a few data points
- Weak interactions unreliable (like Hartree-Fock, semiempirical methods do not describe electron correlation effects that give rise to London dispersion forces, unless some special correction is added like a “-D” dispersion correction)
MNDO/d

- Adds d functions to MNDO
- For metals, these are essential
- For 2\textsuperscript{nd} row atoms, these are helpful in describing polarization
- Can offer significant improvements over MNDO, AM1, PM3
Parameterized Model 6 (PM6)

- NDDO Method with improved parameters and improved core-core interaction term (diatomic parameters like original MINDO)
- 70 elements parameterized
- Corrects some problems with AM1, PM3
- Includes d functions for metals like MNDO/d

Semiempirical Total Energies

Like the Hartree-Fock model they’re derived from, semiempirical methods yield total electronic energies (relative to infinitely separated nuclei and valence electrons)

Unlike HF, we don’t need to add zero-point vibrational energy (ZPVE) when computing enthalpy differences; it’s included *implicitly* in the parameters
### Average $\Delta H_f$ error in kcal/mol

<table>
<thead>
<tr>
<th>Compounds</th>
<th>MNDO</th>
<th>AM1</th>
<th>PM3</th>
</tr>
</thead>
<tbody>
<tr>
<td>All normal valent (607 compounds)</td>
<td>24.3</td>
<td>14.8</td>
<td>11.2</td>
</tr>
<tr>
<td>Hypervalent (106 compounds)</td>
<td>104.5</td>
<td>62.3</td>
<td>17.3</td>
</tr>
<tr>
<td>All (713 compounds)</td>
<td>46.2</td>
<td>27.6</td>
<td>11.6</td>
</tr>
</tbody>
</table>

Data from Frank Jensen, *Introduction to Computational Chemistry* (Wiley, New York, 1999)
## Average $\Delta H_f$ error in kcal/mol

<table>
<thead>
<tr>
<th>Compounds</th>
<th>MNDO</th>
<th>AM1</th>
<th>PM3</th>
<th>MNDO/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al, Si, P, S, Cl, Br, I, Zn, Hg (488 compounds)</td>
<td>29.2</td>
<td>15.3</td>
<td>10.0</td>
<td>4.9</td>
</tr>
<tr>
<td>Si, P, S, Cl, Br, I (404 compounds)</td>
<td>31.4</td>
<td>16.1</td>
<td>9.5</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Data from Frank Jensen, *Introduction to Computational Chemistry* (Wiley, New York, 1999)
### Average $\Delta H_f$ error in kcal/mol

<table>
<thead>
<tr>
<th>Compounds</th>
<th>AM1</th>
<th>PM3</th>
<th>PM6</th>
</tr>
</thead>
<tbody>
<tr>
<td>H, C, N, O (1157 compounds)</td>
<td>9.4</td>
<td>5.7</td>
<td>4.6</td>
</tr>
<tr>
<td>Main group (3188 compounds)</td>
<td>22.3</td>
<td>17.8</td>
<td>6.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Compounds</th>
<th>PM6</th>
<th>B3LYP/6-31G*</th>
<th>HF/6-31G*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1373 compounds</td>
<td>4.4</td>
<td>5.2</td>
<td>7.4</td>
</tr>
</tbody>
</table>

## Average errors in bond lengths (Å)

<table>
<thead>
<tr>
<th>Bonds to</th>
<th>AM1</th>
<th>PM3</th>
<th>PM6</th>
</tr>
</thead>
<tbody>
<tr>
<td>H, C, N, O</td>
<td>0.031</td>
<td>0.021</td>
<td>0.025</td>
</tr>
<tr>
<td>(413 compounds)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H, C, N, O, F, P, S, Cl, Br, I</td>
<td>0.046</td>
<td>0.037</td>
<td>0.031</td>
</tr>
<tr>
<td>(712 compounds)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Main group</td>
<td>0.131</td>
<td>0.104</td>
<td>0.085</td>
</tr>
<tr>
<td>(2636 compounds)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Typical semiempirical accuracy for other properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bond angles</td>
<td>± 4°</td>
</tr>
<tr>
<td>Ionization potentials</td>
<td>± 0.5-1.0 eV</td>
</tr>
<tr>
<td>Dipole moments</td>
<td>± 0.4 D</td>
</tr>
<tr>
<td>Relative energies ($\Delta H_f$)</td>
<td>± 5-10 kcal/mol (not as systematic as ab initio energy errors)</td>
</tr>
</tbody>
</table>
Improvements to PM6 (etc.) for Noncovalent Interactions: PM6-DH, PM6-DH2, PM6-D3H4X

- Add term for dispersion: damped $C_{6,AB}/R_{AB}^6$ terms with same form as in popular −D dispersion corrections to DFT and analogous to London dispersion terms in force fields
- Add term for hydrogen bonding

Parameterized Model 7 (PM7)

- Further improvements upon PM6
- Added explicit terms to describe non-covalent interactions, based on ideas in the −DH2, -DH+, and −D3H4 corrections to PM6 by Hobza and co-workers
- Aimed at giving better results for molecules different than those in the training set
- Corrects two minor errors in the NDDO formalism
- MUE in bond lengths decreased by about 5% and in ΔH\textsubscript{f} by about 10% relative to PM6 (for organic solids, improvement of 60% in ΔH\textsubscript{f})

Modern Semiempirical Methods for Non-Covalent Interactions

Extended Hückel Theory

- Parameterizes the Fock matrix, not the integrals
- $F_{\mu\mu} = -I_\mu$
- $F_{\mu\nu} = -k S_{\mu\nu}(I_\mu + I_\nu)/2$  \hspace{1cm} (k=1.75)
- Very approximate method, but can be useful for getting trends or generating guess MO’s for Hartree-Fock
Neglect of all 3 and 4 center integrals makes formal scaling go from $O(N^4)$ to $O(N^2)$ for construction of Fock matrix.

Diagonalization of $F$ scales as $O(N^3)$ and therefore becomes formal rate-determining step (there are tricks to avoid explicit diagonalization).

Share advantages/disadvantages of force fields: perform well for systems where experimental data available, not reliable otherwise.

Somewhat more robust than force fields because at least based on QM.