CHEM 6491: Quantum Mechanics

Problem Set I

Due Tuesday, September 3

1. Atlanta Braves pitcher John Smoltz can throw a 95 mph fastball. A regulation baseball must weigh between 5 and 5.25 ounces. What is the wavelength of a John Smoltz fastball weighing 5 ounces? (1 lb = 16 oz). Could a researcher detect any wave-like properties (like interference) for fastballs?

2. In Bohr's model of the hydrogen atom, electrons travel around the nucleus in certain stable orbits of fixed radius r. In this case, the centripetal force mv^2/r which keeps the electron from flying away is supplied by the Coulomb attraction, $e^2/(4\pi\epsilon_0 r^2)$. Note that we will assume the electron is orbiting around a fixed nucleus; since the proton is much more massive than the electron (by about 2000 times), this is a good approximation. Otherwise, we could use a reduced mass.

   (a) Equate the centripetal force with the Coulomb force, use the fact that only certain angular momenta are allowed (Bohr's assumption $l = mvr = n\hbar$), and solve for the allowed Bohr orbits r.

   (b) The energy of the hydrogen atom is given by a sum of kinetic and potential energies,

   $$E = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r}. \tag{1}$$

   Again using the relation between the centripetal force and the Coulomb force, rewrite E in terms of r and physical constants only (i.e., eliminate the mv^2 term) and then in the place of r substitute the expression (solved for above) for the allowed orbits. This should give the allowed energy levels.

   (c) Using the fact that a transition between two allowed energy levels is the same as the energy of the photon emitted/absorbed $\Delta E = h\nu$, derive the Rydberg formula for the H atom spectrum,

   $$\Delta E = \frac{me^4}{8\epsilon_0^2 h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) = h\nu, \tag{2}$$

   where n and m are two integers with $n < m$.

3. Which of the following sets of vectors is linearly independent?

   (a) $6\hat{i} + 3\hat{j}$, $-2\hat{i} - \hat{j}$

   (b) $\hat{i} + 2\hat{k}$, $\hat{j}$, $\hat{i} + \hat{j} + \hat{k}$

   (c) $\hat{i} + \hat{j}$, $-\hat{i} + \hat{j}$, $\hat{k}$

(d) $|cos(\omega t)\rangle$, $|2cos(\omega t)\rangle$

(e) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

4. Apply the Gram-Schmidt procedure to $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ to obtain three orthonormal vectors. Begin with the first vector given.